Matrix class-XII

04-07-20

① Matrix :→ A matrix of order m×n is of the form

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Its elements in the i^{th} row and j^{th} Column is a_{ij}.

If $m=n$, the matrix is a square matrix

② Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ are the matrices ; then

(I) Addition of matrix, $A+B = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

(II) Subtraction of matrix, $A-B = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

(III) Multiplication of matrices.

$$A \times B = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

③ If A, B and C are three matrices such that their products are defined, then

(I) $AB \neq BA$ (II) $(AB)C = A(BC)$ (III) $A(B+C) = AB+AC$

④ Transpose of a matrix :-

The matrix obtained by interchanging the rows and columns of a given matrix A, is called its transpose. It is denoted by A' or A^T

⑤ Results on Transpose of matrices

(I) $(A+B)' = A'+B'$ (II) $(kA)' = kA'$ (III) $(AB)' = B'A'$

(IV) $(A')' = A$

Exercise 3.1 Class - XII

Q:1 In the matrix $\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write

(I) the order of the matrix

(II) the number of elements

(III) the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.

Soln: I. In the given matrix, the number of rows is 3 and the number of columns is 4.

∴ The order of the matrix is 3×4.

(II) ∵ the order of matrix is 3×4,

So, there are in 3×4 = 12 elements in it.

(III) Let $\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 19 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

∴ $a_{13} = 19$, $a_{21} = 35$, $a_{33} = -5$, $a_{24} = 12$, $a_{23} = \frac{5}{2}$

Q.2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Sol: We know that if a matrix is of the order m×n, it has mn elements.

Hence, a matrix containing 24 elements can have any one of the following orders

1×24, 2×12, 3×8, 4×6, 6×4, 8×3, 12×2 or 24×1.

Similarly, a matrix containing 13 elements can have ordered 1×13 or 13×1.

Q.3. If a matrix has 18 elements, what are the possible orders it can have? What if it has 5 elements?

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Soln: ∵ a matrix containing 18 elements can have any one of the following orders.

1×18, 18×1, 2×9, 9×2, 3×6, 6×3.

Similarly, a matrix containing 5 elements can have order 1×5 or 5×1.

Q.4. Construct 2×2 matrix, $A = [a_{ij}]$ whose elements are given

(i) $a_{ij} = \frac{(i+j)^2}{2}$ (ii) $a_{ij} = \frac{i}{j}$ (iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Soln (i) The order of the given matrix is 2×2,

so, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$ where $a_{ij} = \frac{(i+j)^2}{2}$

when $i=1, j=1$

$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2$

Similarly, $a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$

$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$ and $a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$

$\therefore A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}_{2\times2}$

(ii) Here, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$, where $a_{ij} = \frac{i}{j}$

$\therefore a_{11} = \frac{1}{1} = 1, a_{12} = \frac{1}{2}, a_{21} = \frac{2}{1} = 2$

and $a_{22} = \frac{2}{2} = 1$

$\therefore A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}_{2\times2}$

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4 (XII) Here, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$ where $a_{ij} = \frac{(i+2j)^2}{2}$

$\therefore a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}$, $a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2}$

$a_{21} = \frac{(2+2)^2}{2} = 8$ and $a_{22} = \frac{(2+4)^2}{2} = \frac{6\times 6}{2} = 18$.

$\therefore A = \begin{bmatrix} \frac{9}{2} & 25/2 \\ 8 & 8 \end{bmatrix}_{2\times 2}$

Q.5 Construct a 3×4 matrix whose elements are given by

(i) $a_{ij} = \frac{1}{2}|-3i+j|$ (ii) $a_{ij} = 2i-j$

Sol'n The order of given matrix is 3×4

$\therefore A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times 4}$, where ~~$a_{ij} = \frac{1}{2}|-3i+j|$~~

where $a_{ij} = \frac{1}{2}|-3i+j|$

$\therefore a_{11} = \frac{1}{2}|-3+1| = \frac{2}{2} = 1$

$a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2}$

$a_{13} = \frac{1}{2}|-3+3| = 0$

$a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2}$

$a_{21} = \frac{1}{2}|-6+1| = \frac{5}{2}$

$a_{22} = \frac{1}{2}|-6+2| = 2$

$a_{23} = \frac{1}{2}|-6+3| = \frac{3}{2}$

$a_{24} = \frac{1}{2}|-6+4| = 1$

$a_{31} = \frac{1}{2}|-9+1| = 4$

$a_{32} = \frac{1}{2}|-9+2| = \frac{7}{2}$

$a_{33} = \frac{1}{2}|-9+3| = 3$

and $a_{34} = \frac{1}{2}|-9+4| = \frac{5}{2}$

$\therefore A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}_{3\times 4}$